Bitterne CE Primary School

Progression of Skills for the Four Operations

Thank you for viewing Bitterne CE Primary School's document for the Progression of Skills for the Four Operations.

This document has been created with the most recent research and is in line with age expectations for the 2014 National Curriculum.

For each subject, the skills are in progressive order. These start with the most basic skills moving up to what the expectations are for a child leaving Year 6.

Contents:

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Version 2.3

Glossary

Brackets	Used to determine the order in which operations are carried out. For example, 3 + 4 x 2 = 11 but (3 + 4) x 2 = 14.
Decimal	Not a whole number – for example: 2.4 or 0.267
Decrease	To make a value smaller
Difference	Subtract the smaller value from the larger value to find the difference between two numbers.
Dividend	The value that is to be divided – e.g. in 15 ÷ 5 = 3, the dividend is 15.
Divisor	The value that determines by how much the dividend will be divided by – e.g. in 15 ÷ 5 = 3, the divisor is 5.
Equal	Used to show two quantities have the same value. The equals sign is = and can be used to show equal values in a variety of different ways. For example: 3 + 2 = 5, 5 = 3 + 2, 2 + 3 = 4 + 1
Estimate	To find an approximate answer to a more difficult problem. E.g. 31.2 x 5.94 is roughly equal to 30 x 6 = 180.
Even Number	Any number which is a multiple of 2. Even numbers always end in 2, 4, 6, 8 or 0.
Factor	A number that divides another number exactly. E.g. 4 is a factor of 12.
Increase	To make a value larger
Multiple	A number which is part of another number's times table. E.g. 35 is a multiple of 5.
Negative	A value less than zero
Odd Number	A number that is not a multiple of 2. Odd numbers always end in 1, 3, 5, 7 or 9.
Positive	A number greater than zero.
Product	The answer when two values are multiplied together.
Square Number	The product when a whole number is multiplied by itself. For example, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.
Remainder	The amount left over when a number cannot be divided exactly. For example, 21 divided by 4 is 5 remainder 1.

For more information…

If you would like further information regarding our Progression of Skills, please see your child's class teacher. For a more specific enquiry, please see one of the Maths team members (Mr. May, Year 6 teacher, Mrs Watts, Year 2 teacher, or Mr. Stocks, Specialist Maths Teaching Assistant) or the Inclusion Leader (Mrs. Bailey, Year 6 teacher).

Please note: the age level expectations are in line with the 2014 National Curriculum. Children will learn strategies that are appropriate to their mathematical understanding, and these will be flexibly used in class.

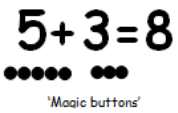
Addition

Year 1:

In Year 1, the priority is to develop a love for maths and number, while building confidence. The initial stages to be built on from Early Years are to ensure that a numeric symbol (e.g. 5) means the same as 5 individual objects. The aim is to develop the child's mental images, which is called subitising. Once this has been developed, the children need to gain more confidence in using arithmetic symbols, such as add (+) and subtract (-), and using these in different concepts.

1.1 The first step of addition is for children to complete **practical problems by** combining two sets of objects. Children would be building on their experience in the Early Years.

This is developed by children drawing using symbols (such as dot patterns) to represent numbers then adding the buttons together. This is only suitable for smaller numbers e.g.



'Magic buttons'

1.2 It is important for children to understand that addition can be done in any order.

e.g. 5 + 3 = 8 **3 + 5 = 8**

The understanding of missing numbers is crucial to developing an understanding of the inverse. This helps children to link their skills for addition and subtraction – e.g.

3 + ____ = 8

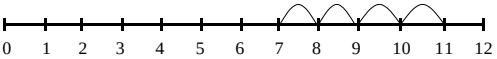
… well to solve this I need to start with 3, and count on to find 8.

1.3 Using children's mental images, they must develop aggregation – combining to sets of objects, which will progress to augmentation - adding on to a set. It is essential that children see these images of dot-patterns, a logical pattern that builds from number to number. (See Inclusion/Maths policy).

1.4 By the end of Year 1, it is expected that children will be recording their addition methods on a number line. Children must learn to count from the largest number, which might mean they need to swap the calculation around independently to start with the largest number.

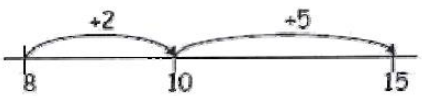
3 + 2 = 5

4 + 7 = 11
Remember to start with the largest value:



1.5 Bridging 10 is an important development gap. Through continuing to develop use of the number line to support addition, this will be achieved. For example:

8 + 7 = 15



Year 2:

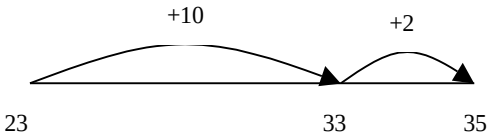
2.1 In Year 2, the use of the equal symbol becomes extremely important. Children must be confident by to solve missing number problems - for example:

16 + 5 = 10 + □ **50 + □ + □ = 100** **30 = 1 + □ + 5**

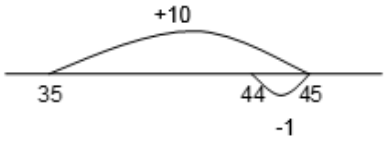
Some of these problems can be "open" – i.e. a variety of different correct answers are possible. Tackling these "open" problems are extremely important in building a child's confidence in maths.

2.2 When children begin to add two-digit values, they will have to have this secure understanding of place value established. This involves using place value knowledge to count in tens and ones.

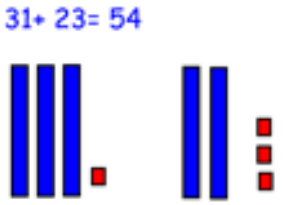
e.g. 23 + 12 –
23 plus 10 to make 33
Then 33 plus the 2 to find the answer.



2.3 Alongside this, children must develop their skills in manipulating numbers. One of these skills is called "adjusting". When adding values such as 9, 19, 29 etc., children can find this tricky. However, by simply adding 10 and subtracting one, this helps the child to find the answer using skills they are more confident with.



2.4 In Year 2, children begin to use dienes blocks a lot to develop their mental understanding of addition. Children will create the number using dienes blocks by partitioning the tens and ones. They will line them up side by side, before combining both numbers together and counting the tens and ones. Where children have more than 10 ones, they will use their skills of exchanging. For example, if there are 12 ones left over, children will visit the "bank" to exchange 10 ones for a ten, and then have ten and 2 ones.



2.5 The final written step in Year 2 is to move onto a formal written method. Building on their knowledge of deines blocks, children will use the tens and ones to add separately, recording it in a similar form to the left. This is the beginning of children mastering the "columnar written method", which the children are expected to be fluent in by the end of Year 3.

$$\begin{array}{r} 40 + 7 \\ +\ 20 + 5 \\ \hline 60 + 12 = 72 \end{array}$$

2.6 Although children may move onto 3-digit numbers (hundreds, tens and ones), it is much better for them to master their methods and become really confident in encountering methods in different forms. This might involve missing values – for example:

$$\begin{array}{rr} & \_\_?\_\_ + 8 \\ + & 20 + 3 \\ \hline & 60 + \_?\_ = \_\_?\_\_ \end{array}$$

These missing number problems use the child's mental maths and reasoning to work out where to begin, causing them to think carefully about the steps they have to take.

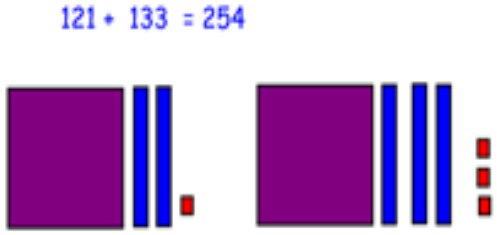
Year 3:

Year 3 builds on the skills taught in Year 2, but with larger numbers.

3.1 Partitioning is still a key skill, although this is done more mentally with jottings, rather using a formal method. It is hoped that later in the school, children will complete these with a minimum amount of jottings and be very efficient mentally.

e.g. 451 + 325 =
451 + 300 = 751
751 + 20 = 771
771 + 5 = 776

3.2 Children will become more efficient with expanded methods, and use larger numbers. Dienes blocks will still form an integral part of their visual understanding.



3.3 Partitioning in columns – Expanded Written Method

Building on from Year 2, children will learn the expanded method but using larger numbers. This time children are expected to start from the right, with the ones. This is very important because it introduces children into the notion of starting from the right with all types of operation.

475 + 354 =

	400	70	5
+	300	50	4
	700	120	9

700 + 120 = 820
→ 820 + 9 = 829

3.4 Stacking beginning from the right (ones)

As children become more confident in this method, children can then move to adding the least significant value first. This means that when they move onto the full column method, they are familiar in starting from the least significant digits from the right hand side.

536 + 307 =

	536	
+	307	
	13	6 + 7
	30	30 + 0
	800	500 + 300
	843	

3.5 Standard Column Method

475 + 323 =

```
  4 7 5
+ 3 2 3
  7 9 8
```

Steps to success:

1) Start from the right. Add the ones values (5 + 3 = 8)
2) Add the tens. (70 + 20 = 90). Children must understand that, although it looks like they are completing 7 + 2, they are actually completing 70 + 20. Their place value knowledge from prior partitioning is crucial here).
3) Add the hundreds. (400 + 300 = 700). Children must understand that, although it looks like they are completing 4 + 3, they are actually completing 400 + 300. Their place value knowledge from prior partitioning is crucial here).

3.6 When crossing thousands, hundreds and tens:

783 + 458 =

Step 1:

```
  7 8 3
+ 4 5 8
    1
  1
```

Step 2:

```
  7 8 3
+ 4 5 8
  4 1
1 1
```

Step 3:

```
  7 8 3
+ 4 5 8
1 2 4 1
  1 1
```

Steps to success:

1) Start from the right. Add the ones values (3 + 8 = 11). The one value 1 goes in the ones column, with the ten underneath the tens column.

2) Add the tens (80 + 50 = 130). Children must understand that, although it looks like they are completing 8 + 5, they are actually completing 80 + 50. Then children must add the ten they have carried over – 80 + 50 + 10 = 140.
Children write the 4 in the tens column and the 1 under the hundreds column.

3) Add the hundreds. (700 + 400 = 1100). Then add the extra hundred carried over – 1100 + 100 = 1200. Children would write the 2 in the hundreds column to represent 200, and then write 1 in a new thousands column to signify the 1000 in 1200.

Year 4 and beyond:

No new methods are taught from Year 4 onwards – it is all about mental efficiency and increasing difficulty. By the end of year 4, children are expected to be fluent in adding values up to four digits, and also values to 1 decimal place.

Mental partitioning is extremely important. From Year 5, children are expected to solve calculations using partitioning very efficiently. An example might be:

15563 + 2400 =

15563 + 2000 = 17563

17563 + 400 = 17963

Throughout Key Stage 2, it is expected that children meet the column method in various scenarios. Problem solving activities such as leaving out missing numbers, giving wrong answers that require mistake spotting, and looking for rules/patterns are essential.

Subtraction

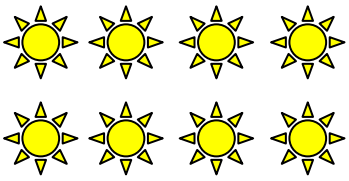
In Year 1, the priority is to develop a love for maths and number, while building confidence. The aim is to develop the child's mental images, which is called subitising. Once this has been developed, the children need to gain more confidence in using arithmetic symbols, such as add (+) and subtract (-), and using these in different concepts.

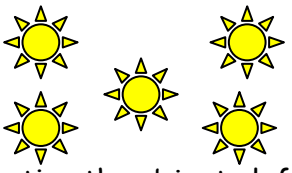
1.1 For the early stages of subtraction, it is really important that again children explore the concept in the world around them. Children must start thinking about 1 less than, 2 less than and 10 less than. Counting backwards is also an incredibly important skill to develop, alongside counting forwards.

1.2 By the end of year 1, children are expected to be able to calculate simple calculations using pictures or practical resources, and write these down. Children will learn best initially by using physical objects to take away. Numbers below 10 should be shown as dot patterns. When bridging 10, the caterpillar track should be used to help children to understand what 10 means.

For example: 8 – 3

Start with 8 objects and then physically remove three of them.

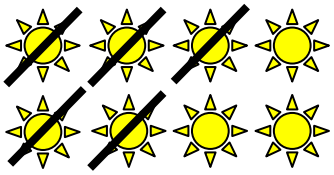




Counting the objects left

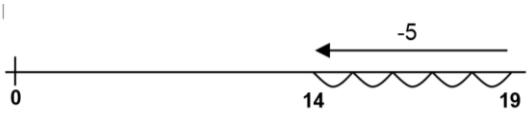
Removed:

This can also be represented by crossing off objects e.g.



1.3 <u>Using a number line</u>

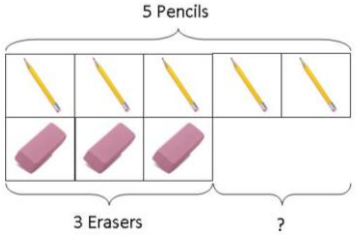
When children are confident in their understanding, they can begin to demonstrate their understanding on a number line.



At Bitterne CE Primary School, when taking away, we do this underneath the number line. This is to show that our calculation is different to an addition calculation, and show that the number is getting smaller. Initially, we would count back in ones.

1.4 Children must gain confidence in identifying missing values in calculations, such as 9 - ___ = 4

Before children understand how to solve these calculations, they must see the difference visually:



Real life problems are essential to develop understanding.

How many more pencils have I got than erasers?

1.5 There are many intricate stages in progression when subtracting. Each stage is important to a child's understanding of subtraction.

Progression of skills	**Red – Year 1 focus**	**Blue – Year 2 focus**
Single digit minus 1	e.g.	7 – 1 =
Single digit minus another one digit number	e.g.	7 – 3 =
Two digit number minus a one digit number, not crossing the ten	e.g.	15 – 3 =
10 minus a one digit number	e.g.	10 – 2 =
20 minus a one digit number	e.g.	20 – 6 =
A two digit number minus 10	e.g.	17 – 10 =
A two digit number minus a one digit number, crossing the ten	e.g.	12 – 5 =
A two digit value minus 10	e.g.	17 – 10 =
A two digit value in the tens minus another multiple of 10	e.g.	40 – 20 =
A two digit value minus another multiple of 10	e.g.	45 – 20 =
A two digit value minus another two digit value - no exchanging	e.g.	45 – 23 =
A two digit value minus another two digit value - exchanging	e.g.	43 – 25 =

Year 2

2.1 Understanding missing number calculations are very important in developing a child's confidence in maths, and their ability to manipulate number. When they are more confident with subtraction, they can use the inverse with addition to solve increasingly complex problems – for example:

52 - 8 = ____

____ - 30 = 25

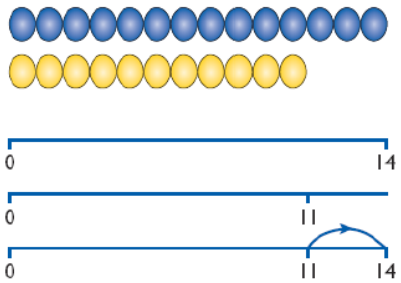
30 = ____ - 10

8 + ___ + 7 = 20

As with addition, children must gain a deep understanding of what = means in maths.

2.2 Difference and Take Away

Taking away is a separate skill to finding the difference. Difference is used when two numbers are fairly close together, and it would make more sense to count on from the smaller value to find the larger value.



The difference between 11 and 14 is 3.

$14 - 11 = 3$

$11 + \square = 14$

Children must develop an understanding of difference. They must be able to recognise whether it would be quicker to find the difference or take away. For example, 52 – 3 is a subtraction, but with 52 – 49, children must understand to count on from 49 to reach 52 – therefore, finding the difference. Children will develop a greater understanding of difference throughout Year 3.

2.3 Year 2 begin by using their mental calculations to break more complex calculations down into easier steps. An example of this might be to split a number when bridging 10.

12 – 5 =

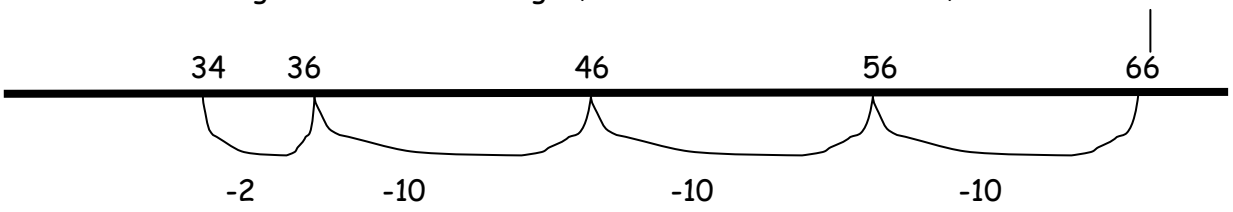
Step 1 : Find the nearest value of 10 **12 – 2 = 10** Step 2 - Subtract the remaining value **10 – 3 = 7**

2.4 <u>Empty Number Line to take away values</u>

As with addition, children use their knowledge of partitioning to help them solve subtraction calculations with larger values. Children must learn to subtract first the tens, than the ones.

66 - 32

Place the highest value to the right, and subtract in tens first, then the ones.

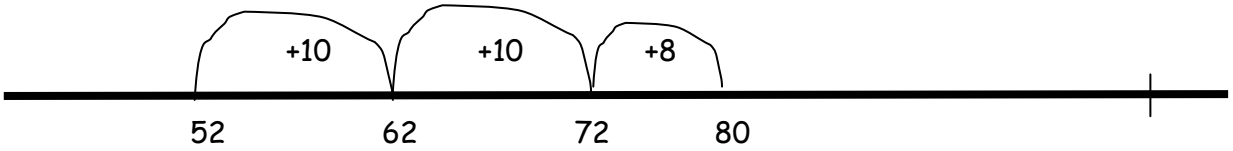


Children then add up the values they have subtracted → 10 + 10 + 10 + 4 = 34

When children are more confident they can combine the jumps of 10 to simply subtract 30.

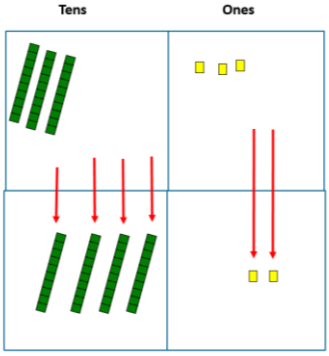
Finding the difference is where children must find a missing value to calculate the answer. For example:

52 + ____ = 80 As we are adding, the values are above the line.



Children then add up the values to find the difference → 10 + 10 + 8 = 28 – The difference is 28.

2.5 The final written step in Year 2 is to move onto a formal written method. Building on their knowledge of deines blocks, children will use the tens and ones to subtract separately, recording it in a similar form to the left. This is the beginning of children mastering the "columnar written method", which the children are expected to be fluent in by the end of Year 3.



	70	5
-	40	2
	30	3

At this point, children may be introduced to this skill alongside the skill of exchanging, but they may not yet be combined. (Exchanging is where a child would need to exchange a ten for 10 ones, when they do not have enough ones to subtract).

Year 3

As with addition, by the end of Year 3, children are expected to use the standard column method to subtract up to three digits. Children would initially apply their knowledge using the empty number line to calculations that involve three digits. However, by the end of Year 4, children are expected to use the standard column method to subtract up to four digits.

3.1 Partitioning in columns

346 – 124 =

Children must start from the right

	300	40	6
-	100	20	4
	200	20	2

Children then add the relevant sections together: 200 + 20 = 220 → 220 + 2 = 222

3.2 Although children may move onto 3-digit numbers (hundreds, tens and ones), it is much better for them to master their methods and become really confident in encountering methods in different forms. This might involve missing values – for example:

	70	+	8		
-	?	+	3		
	40	+	**?**	=	**45**

3.3 Partitioning in columns, but using exchanging

A common misconception is where children just reverse the numbers to solve a problem – for example, for 2 – 4, they will simply change it around to 4 – 2. This is wrong, and we must ensure children do not do this.

347 – 164 =

Children must start from the right.

In this calculation, we exchange a hundred with 10 tens and make the 40 → 140.

	200		
	~~300~~	140	7
-	100	60	4
	100	80	3

Children then add the relevant sections together: 100 + 80 + 3 = 183

3.4 The next step is where calculations have more than one value to exchange from → **412 - 167 =**

Children must start from the right.

	300	100	
	~~400~~	~~10~~	12
-	100	60	7
	200	40	5

* Children cannot complete 2 – 7 so must exchange a 10
* Zero tens are left. Therefore, we must exchange one hundred with ten 10s to complete out next step.
* As we have stolen one hundred, we must complete 300 – 100.
* Children then add the relevant sections together: 200 + 40 + 5 = 245

3.5 Standard Column Method for Subtraction

When children are confident in this step, they can move onto the standard written method for subtraction. By the end of Year 3, children are expected to solve subtraction calculations up to three digits confidently.

412 – 167

```
  3  10  1
  4  1  2
- 1  6  7
  2  4  5
```

When children progress to the compact method, it should be seen as a continuation of the expanded method in streamlined form, rather than a new method.

Year 4 and beyond

No new methods are taught from Year 4 onwards – it is all about mental efficiency and increasing difficulty. By the end of year 4, children are expected to be fluent in subtracting values up to four digits, and also values to 1 decimal place. Children are also expected to be able to exchange at least once in a calculation.

Mental partitioning is extremely important. From Year 5, children are expected to solve calculations using partitioning very efficiently. An example might be:

15563 - 2400 =

15563 - 2000 = 13563

13563 - 400 = 13163

Throughout Key Stage 2, it is expected that children meet the column method in various scenarios. Problem solving activities such as leaving out missing numbers, giving wrong answers that require mistake spotting, and looking for rules/patterns are essential.

Throughout the school, we teach children extra strategies that may help them solve calculations.

Using their knowledge of adjusting and difference…

```
  5 0 0 2          → subtract 3 →          4 9 9 9
- 4 5 3 4          → subtract 3 →        - 4 5 3 1
_________                                  0 4 6 8
```

Here, I have subtracted three numbers to remove the need to exchange. The difference stays the same between the numbers; therefore, the answer will be the same.

The following strategy is especially useful when solving money problems as children progress up through the school.

£10.00 - £5.75

→ would become £9.99 - £5.74

Both numbers have 1p subtracted from them.

```
  9 . 9 9
- 5 . 7 4
  4 . 2 5
```

The difference between the values for each calculation is £4.25.

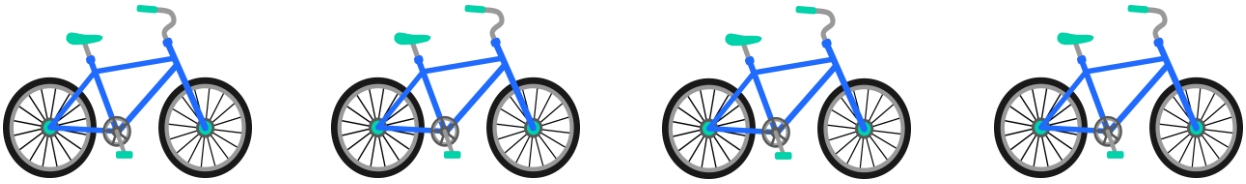
Children must remember the value they are working in. For example, if children are working in money, they must have an answer only to two decimal places.

Where children are completing calculations with decimals of different length, they must include place holders (extra zeros) to ensure the decimals are the same length.

Multiplication

Year 1

1.1 The early stages of multiplication simply involve counting up in the multiple (counting up in the times table value). For example, this could be counting aloud in multiples of 2, 5 and 10s. Practical objects are very important to support the learning, such as counting pairs of socks, fingers on gloves, wheels on bikes etc.



2 wheels, 4 wheels, 6 wheels, 8 wheels…

It is extremely important that children develop an understanding of multiplication as **repeated addition**. Children need to understand that multiplication is adding the same number together multiple times, such as 2+2+2+2 means 2 x 4 (2 four times). Use of concrete resources will support this understanding e.g. 4 **lots of** 2.

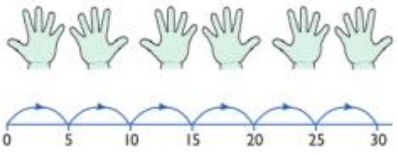
1.2 Arrays are visual representations of a number, which are then represented multiple times to reflect the process of multiplication. Children begin to understand that multiplication is **commutative**, which means that it can be carried out with the numbers either way around, e.g. 4x6 is the same as 6x4. It is important to share the vocabulary with the children. The introduction of **arrays** helps children to see this.

1.3 The diagram below illustrates how arrays can be used to demonstrate how the calculations are different.

5 x 6 = 30

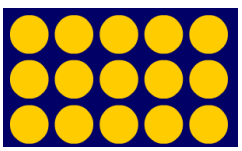
Number in a group	Number of groups

5 finger x 6 hands = 30



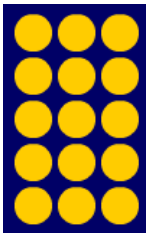
5 + 5 + 5 + 5 + 5 + 5 = 30
5 × 6 = 30
5 multiplied by 6
6 groups of 5
6 hops of 5

1.4 The use of arrays is also extremely important in developing a child's visual understanding of multiplication: **Number in a group** **Number of groups** **Total**



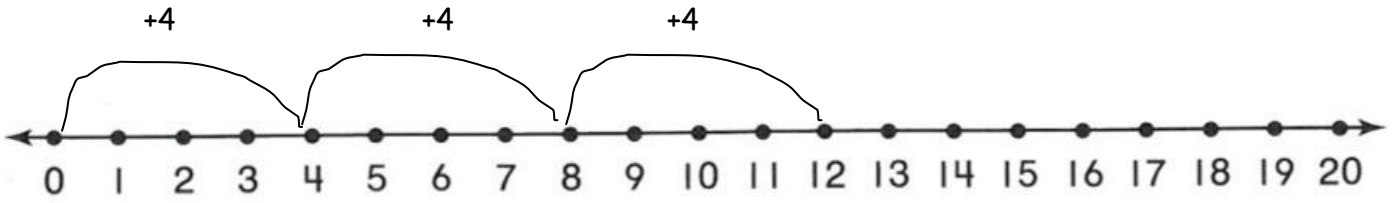
← 3 x 5 = 15
There are 5 groups of 3.

5 x 3 = 15 →
There are 3 groups of 5.

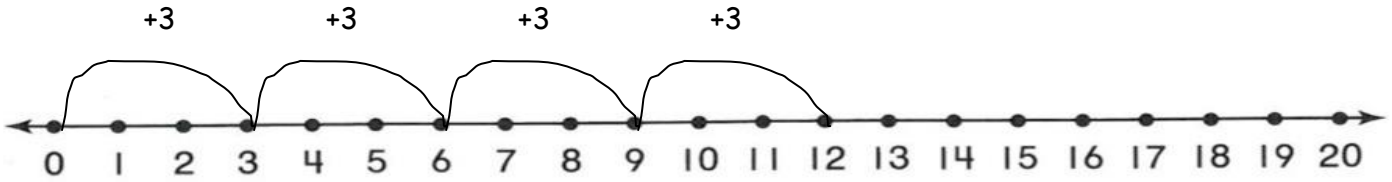


1.5 We can represent this on a number line.

4 x 3 = 12 (3 lots of 4) - "4 three times" "3 lots of 4" "4 + 4 + 4"



3 x 4 = 12 (4 lots of 3)



When children are confident in using this method, they will then move to use the empty number line to solve multiplication problems.

Year 2

2.1 In Year 2, children become more independent in recording their own calculations. They must express multiplication calculations as a number sentence using x.

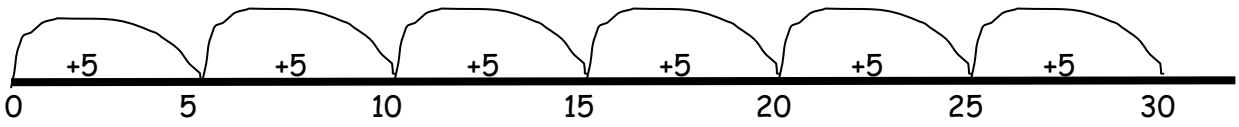
As children gain confidence, they must develop their understanding of using the inverse and practical resources to solve missing number problems.

7 x 2 = □	**□ = 2 x 7**
7 x □ = 14	**14 = □ x 7**
□ x 2 = 14	**14 = 2 x □**
□ x ◯ = 14	**14 = □ x ◯**

Again, children must develop their understanding of the = symbol, and the different roles this can play in a number sentence.

2.2 Children will continue to build on their Year 1 knowledge of arrays and number lines. Dienes blocks will be used alongside the number line to help reinforce the mental image.

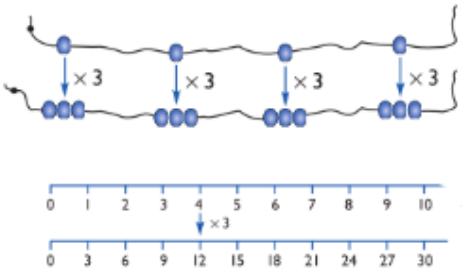
5 x 6 = 30



When children are confident in this method, they will then move to using the empty number line for multiple jumps.

5 x 12 = 60

2.3 Children will develop their knowledge of scaling.



The most effective way for this to develop is through real-life, practical problems.

For example;

If 1 car has 4 wheels, how many wheels will 5 cars have?

This is an essential step in understanding multiplication. It is also very important that children are supported when solving these problems with practical objects, as this will help deepen their understanding. A deep understanding in scaling is crucial to understanding ratio, proportion, fractions and other concepts when the child is older.

2.4 By the end of Year 2, it is expected that children will have made the first steps to recording their multiplication beyond the number line. They may make jottings for doubling two digit numbers – such as double 17:

Double 17 → the same as 17 x 2

17 x 2 = → 10 x 2 = 20 → 20 + 14 = 34

17 x 2 = → 7 x 2 = 14 → 20 + 14 = 34

As illustrated, these jottings would partition the two digit values. Then, the child would complete the multiplication problem using numbers and calculations they are more confident with.

Year 3

3.1 By the end of year 3, children are expected to count forwards and backwards in 10s, 5s, 2s, 3s, 4s, 8s, 50s and 100s. They are expected to know and use the multiplication and division facts for the 10x, 5x, 2x, 3x, 4x and 8x tables. Using the strategies learnt in Year 2, children would be expected to multiply values mentally using partitioning with increasing efficiency.

e.g. 15 x 4 → 10 x 4, then 5 x 4, then add the answer.

3.2 During year 3, children will be exposed to more formal and efficient ways of multiplying values. This is through using the '**grid method**'. Children are given opportunities to master this method using dienes blocks.

63 x 4 =

For this calculation, we partition each value into the place value (here in tens and ones, but more difficult calculations would be HTU or ThHTU etc.).

x	60	3
4	240	12

Step 1 Complete 60 x 4. Children are expected to complete this mentally. One way could be to complete 6 x 4 = 24, then x10.

Step 2 Complete 3 x 4

Step 3 Add together 240 and 12 → 240 + 12 = 252

Year 4

4.1 By the end of year 4, children will be expected to know and use the multiplication and division facts for the all the previously mentioned tables, plus 6x, 7x, 9x, 11x and 12x, as well as being able to count confidently in multiples of 25 and 1000.

Their knowledge will build on the equations learnt from year 2, but introduce missing numbers – e.g. ___ **6 x 4 = 64**
Before solving the problem, we would discuss which value was most likely to be the tens.

4.2 Children will continue to master the grid method using dienes blocks. They will build to larger values with more steps.

363 x 5 =

x	300	60	3
5	1500	300	15

Step 1	Complete 3 x 5	
Step 2	Complete 60 x 5	Children are expected to complete this mentally. One way could be to complete 6 x 5 = 30, then x10.
Step 3	Complete 300 x 5	Children are expected to complete this mentally. One way could be to complete 3 x 5 = 15, then x100
Step 4	Add together 1500 + 300 = 1800 and then 1800 + 15 = 1815.	

247 x 23 = 5681

x	200	40	7
20	4000	800	140
3	600	120	21

Step 1	Complete 7 x 20	
Step 2	Complete 7 x 3	
Step 3	Complete 40 x 20	Children are expected to complete this mentally. One way could be to complete 40 x 2 = 80, then x10.
Step 4	Complete 40 x 3	Children are expected to complete this mentally. One way could be to complete 2 x 7 = 14, then x10.
Step 5	Complete 200 x 20	Children are expected to complete this mentally. One way could be to complete 20 x 2 = 40, then x100.
Step 6	Complete 200 x 3	Children are expected to complete this mentally. One way could be to complete 2 x 3 = 6, then x100.
Step 7	Add all of the answers	

Year 5

5.1 In Year 5, children will explore more complex mental methods and rules that will support them in their knowledge of written methods.

- Multiplying and dividing by 10, 100 and 1000
- Identifying patterns of factors, and prime numbers within these
- Finding factor pairs for different values

5.2 During year 4, children may become familiar with more standard, efficient written methods for multiplication. This becomes the expectation in Year 5.

62 x 6

```
  6 2
x   6
-----
  1 2   ← Here children complete 6 x 2
+3 6 0  ← Here children complete 6 x 60
-----
  3 7 2
-----
```

These extra steps in the written method help children to apply their knowledge of partitioning that they had learned when using the grid method.

72 x 34 = 2448

```
    7 2
  x 3 4
-------
      8   ← Here children complete 4 x 2
+   2 8 0 ← Here children complete 4 x 70
+     6 0 ← Here children complete 30 x 2
+ 2 1 0 0 ← Here children complete 30 x 70
---------
  2 4 4 8
---------
    1
```

These extra steps in the written method help children to apply their knowledge of partitioning that they had learned when using the grid method.

5.3 As children become more efficient in completing the written column method for multiplication, they can begin to simplify it and remove steps. It is important that all children see these methods as part of the same method. For example:

6 2 ← children must start from the right

```
  x   6
-------
  3 7 2
-------
    1
```

5.4 Children will then move onto **long multiplication.**

```
          7  2
x         3  4
-------------
      2   8  8
+ 2   1   6  0
-------------
  2   4   4  8
-------------
      1
```

Children start from the right. They begin by multiplying 2 x 4 and 70 x 4 as they do in the simplified short method.

Here we have 72 x 4

As children are multiplying by 30 and not 3, they must remember to put in the zero to signify this. Then complete **2** x **3**0 and **7**0 x **3**0. Because the zero has been added, children can focus simply on the tables they know mentally.

Year 6

6.1 By the end of Year 5, it is expected that children can complete 4-digit values multiplied by 2-digit values.

Here is an example of 2457 x 17. The answer of 2457 x 17 = 41769.

		2	4	5	7	
x	x			1	7	Children start from the right. They begin by multiplying by multiplying 2457 x 7.
	1	7	1	9	9	Here we have 2457 x 7
		3	3	4		Children carry over the place value into the next column to the left.
+	2	4	5	7	0	Here we have 2457 x 10. Don't forget the 0 to show that the calculation is all multiplied by 10.
	4	1	7	6	9	Then add 2457 x 7 and 2457 x 10 together.
	1		1			

6.2 Multiplying decimals is a skill required by Year 6 students, up to 2 decimal places.

Multiplication

32.075 x 4 =

→ Ignore the decimal point – but remember that the number is to 2 decimal places.

			3	2	0	7	5
		x					4
		1	2	8	3	0	0
					3	2	

The question contained a number to 3 decimal places, so this answer is 128.300

There are a number of ways to remember how to do this.

If there is only one value, you could simply remove the decimal point, and remember to put it back into the answer.

If the value was to 3dp in the question, it will be in the answer.

Multiplication

32.075 x 4 =

			3	2	0	7	5
		x		4			4
		1	2	8	3	0	0
					3	2	

The question contained a number to 3 decimal places, so this answer is 128.300

Alternatively, children can leave the decimal point in the first number, but remember to multiply every value by 4.

The 4 can be placed **at the end** or in the **correct ones place**, but children must remember **not** to put in place holders after the decimal point and multiply by those place holders.

Multiplication

32.075 x 2.4 =

3 dp 1 dp the answer needs 4dp

			3	2	0	7	5
		x				2	4
							0

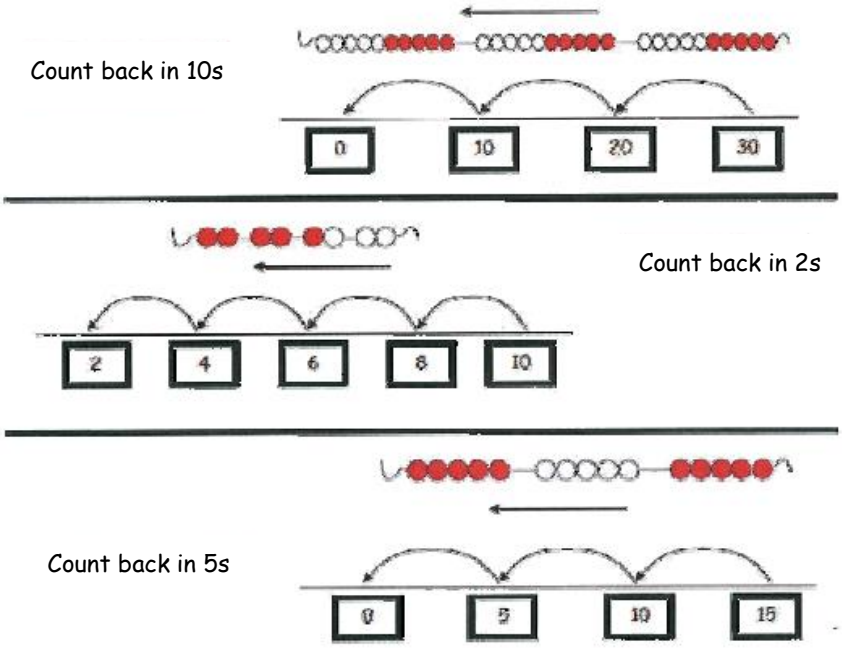
When there are two decimal numbers in the question, first we must see how many decimal places there altogether. The answer is 4 (3dp in 32.075 and 1dp in 2.4). Therefore, the answer requires a value to 4 decimal places.

We advise students to remove the decimal places and treat the calculation then as a long multiplication calculation, but asking them to remember the answer is to 4 decimal places!

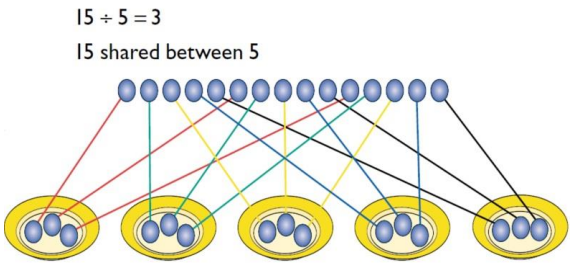
Division

Year 1

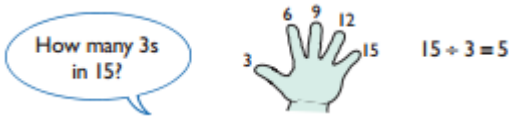
1.1 The early stages of division are similar to those of multiplication – children must become increasingly efficient in counting aloud in multiples of 2, 5 and 10s. Children should aim to be confident in counting forwards and backwards in the multiple, with the values increasing and decreasing.



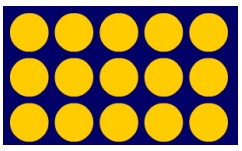
1.2 Children need to experience **'equal-sharing between'** with practical objects before any recording is introduced. This should be referred to as 'equal-sharing between' and not just 'equal-sharing' or 'equal-sharing with'. Children should be taught using physical resources.



1.3 Children should apply their counting skills to help them develop an understanding in the concept of grouping, initially starting with 2s, 5s and 10s, but moving onto 3s, 4s and 100s when ready.

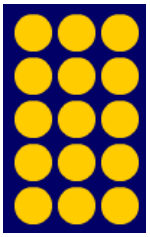


1.4 The use of arrays is also extremely important in developing a child's visual understanding of division: **Total** **Number of objects within a group** **Number of groups**



← 15 ÷ 3 = 5
There are 5 groups of 3.

15 ÷ 5 = 3 →
There are 3 groups of 5.



Year 2

2.1 Children should build on the visual and practical work with grouping and sharing that they have started in Year 1. Children should be introduced to the division (÷) sign and increasingly record their own number sentences.

2.2 As with the other operations, children should begin to experiment with missing number calculations. They should use their knowledge of multiplication to solve problems. Children will need to use the **inverse**:

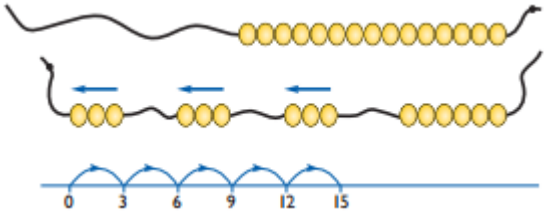
8 ÷ 2 = □ **□ = 8 ÷ 2**

8 ÷ □ = 2 **4 = 8 ÷ □**

□ ÷ 2 = 4 **4 = □ ÷ 2**

□ ÷ ∇ = 4 **4 = □ ÷ ∇**

2.3 Children should build on their knowledge of grouping to use a number line.

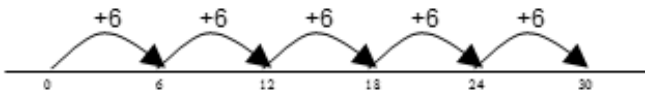


Here, the calculation 15 ÷ 3 = 5 is completed. **Starting from 0**, children would **count** in groups of 3 to find out how many groups they have in total.
Using a **bead string**, children would initially start with 15 beads, before taking off groups of 3. They would count out how many groups they would have in total at the end. **Practical objects** should continue to be used, as well as teaching the children how to draw these jumps on a number line. Empty number lines should be used for children who are more confident.

Year 3

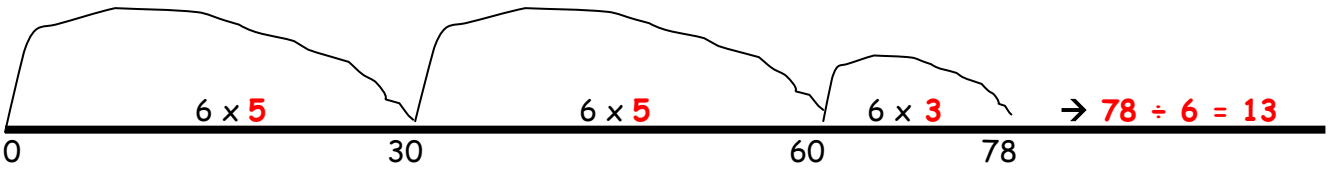
3.1 Children should continue to manipulate equations with missing numbers, becoming confident in the strategies to find the inverse values. Again, open equations should be used at times (e.g. **□ ÷ ∇ = 4**) to give the children experience in finding different answers to fit the rule.

3.2 Children would continue to use the empty number line, starting from zero and counting up.
30 ÷ 6 = 5



Count above the line as the value increases.

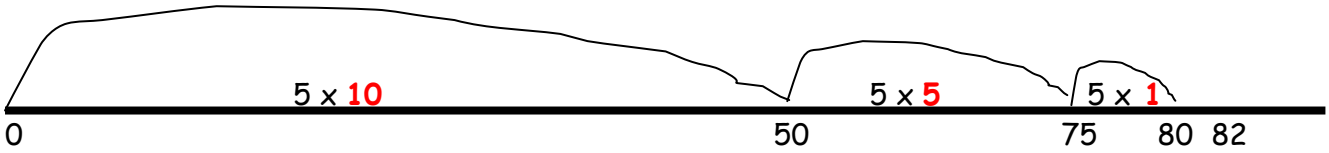
3.3 Chunking on a number line: Chunking is a more efficient method of calculating division using an empty number line. It uses children's knowledge of their tables to help make fewer jumps, thereby speeding up the process. For example: **78 ÷ 6…**



Children would take the divisor (here it is **6**). They would multiply the divisor by values they are confident with – i.e. 1, 2, 5 and 10 – until they reach the total value.

3.4 Remainders could occur. Children would need to be very confident that their jumps

For example: **82 ÷ 5…**



Here, children can only count to 80 in groups of 5. Between 80 and 82, the difference is 2. Therefore, a group of 5 is too large. Therefore, the answer: **82 ÷ 5 = 16 remainder 2**

3.5 Children are expected to begin to develop their knowledge of division using place value.
For example: **80 ÷ 10 = how many groups of 10 in 80?**
800 ÷ 100 = how many groups of 100 in 800?

Year 4

4.1 Initially, children will continue to build on their knowledge of sharing and grouping from Years 2 and 3. Children will continue to represent calculations on a number line until they have a secure understanding.

The numbers will become more complex and in line with the expectations of the curriculum:

- Calculations using tables facts they are fluent with
- Experiencing a logical progression in the numbers they use – e.g.:
  - Using the dividend (the total number) just over 10x – e.g. 72 ÷ 6 =
  - Applying this to just over 100x – e.g. 720 ÷ 6 =
  - Applying the 10x value to a 20x value – e.g. 144 ÷ 6 =

Children will work with remainders in the context of the question. Remainders should be interpreted according to the context within the question, which may mean they are to be rounded up or down.

4.2 Children will begin to chunk vertically, building towards long division – e.g. **72 ÷ 5 =**

The divisor is 5	1	4	remainder 2
5	7	2	
-	5	0	Use chunking to subtract chunks of the divisor 72 – (5 x **10**)
	2	2	
	2	0	Use chunking to subtract remaining chunks. The child can use their knowledge of multiplication tables to assist them. Here 22 – (5 x **4**)
		2	If the answer here is 0, there is no remainder. If there is an answer here which cannot be divided further by 4, this is the remainder.

Children then count up the number of times they have subtracted the divisor – here there are 14.

4.3 Children can apply this strategy to larger values – e.g. **136 ÷ 6 =**

		2	2	remainder 4
6	1	3	6	
-	1	2	0	Children choose a value to subtract. For example: **20** x 6 = 120
		1	6	
-		1	2	2 x 6 = 12
-			4	

Count up the number of times you have subtracted the divisors – here 22. Apply the remainder (if there is one).

Year 5

5.1 Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines previously). Children are expected to interpret remainders in context of worded problems, including as a **fraction** or **decimal**.

98 ÷ 7 becomes

```
  1 4
  ---
7 9 8
    2
```

Answer: 14

432 ÷ 5 becomes

```
    8 6 r 2
    -----
5 4 3 2
    3
```

Answer: 86 remainder 2

496 ÷ 11 becomes

```
    4 5 r 1
    -----
1 1 4 9 6
    5
```

Answer: $45\frac{1}{11}$

Year 6

6.1 In Year 6, children are expected to solve calculations where 4 digits are divided by 2 digits fluently, using both the short and long division method.

432 ÷ 15 becomes

```
      2 8 r 12
      --------
1 5 | 4 3 2
    - 3 0 0
      -----
      1 3 2
    - 1 2 0
      -----
        1 2
```

Answer: 28 remainder 12

432 ÷ 15 becomes

```
      2 8
      ---
1 5 | 4 3 2
    - 3 0 0   15×20
      -----
      1 3 2
    - 1 2 0   15×8
      -----
        1 2
```

$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

```
      2 8 · 8
      -------
1 5 | 4 3 2 · 0
    - 3 0   ↓
      -----
      1 3 2
    - 1 2 0   ↓
      -------
        1 2 0
      - 1 2 0
        -----
            0
```

Answer: 28·8

6.2 Children are expected to confidently divide values with up to 2 decimal places – for example: **5.75 ÷ 5 =**

	1	.	1	5
5	5	.	7	25